P: ISSN NO.: 2394-0344 RNI No.UPBIL/2016/67980 VOL-4* ISSUE-1* April- 2019
E: ISSN NO.: 2455-0817 Remarking An Analisation

Unsteady Flow of Viscous Fluid through A Straight Long Porous Circular Pipe Due to Time Varying Pressure Gradient with an Initial Arbitrary Velocity Distribution

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Abstract

In this paper, we have studied the unsteady motion of viscous fluid through a straight porous channel due to pressure gradient with an initial arbitrary velocity distribution. The fluid is assumed to be Newtonian and incompressible. The technique of finite Hankel transformation has been used to solve the problem. The method, we have applied, put some restriction on injection parameter that it can not be ≤ -2v, but in case of suction there is no such restriction.

Keywords: Viscous, Gradient, Arbitray

Introduction

One dimensional flow problems involving porous medium have important application in various disciplines like petroleum engineering, Geo-Physics and Agricultural engineering etc. The study of blood flow between two permeable layers is of considerable importance in Biomechanics.

The problem of unsteady flow of viscous incompressible fluid in an annulus of two porous co-axial circular cylinders subjected to suction or injection has been studied by Rao [1961] under the presence of a periodic pressure gradient. Singh [1967] has studied the flow of visco-elastic Maxwell fluid in the annulus of two porous concentric circular cylinders under the influence of pressure gradient. The problem of flow through straight channel with an arbitrary time varying pressure gradient and an arbitrary initial velocity has been considered by Das and Goswami [1986]. The problem of flow of a viscous incompressible fluid between two parallel plates, one in uniform motion and the other at rest with uniform suction at the stationary plate has been solved by Verma & Bansal.

In the present paper, we have solved the problem of unsteady flow of viscous incompressible fluid through a straight porous channel due to pressure gradient with an arbitrary initial velocity distribution.

Aim of the Study

In the remainder of this study, conditions corresponding to use with two nonvanishing harmonic components. This is done in the interest of clarity, owing to the large number of choices that may be suitable for discussion. The evolution of the velocity given by Eqs. (1-4) and (5) is illustrated. To proceed, the fundamental equations will be simplified by Eqs., introducing the assumptions of fluid incompressibility and axial flow. The absence of a transverse velocity component leads to a fully developed profile and the cancellation of nonlinear convective terms. This immediate simplification permits the superposition of temporal features associated with pulsatory motions on the steady and fully developed channel flow.

Review of Literature

The fluid dynamics of periodic flows, analysis exposes several characteristic parameters that can be used in a variety of technological applications. These include the development of precise control mechanisms (such as injectors or electronically actuated valves) in studies involving pulse flow velocimetry, laminar-to-turbulentflow transition, fuel injector optimization, mechanically assisted respiration, reverse osmosis, and acoustic wave propagation. Work in this direction is already underway, as some of the parameters described here are being considered in guiding and planning controlled experimental and numerical investigations of pulsatory flow with prescribed pressure or mass flow rates. Examples include those carried out by Ray et al. [1928], Ünsal and Durst [2006], Ünsal et al. [2005], and Durstet al. [2007].

Formulation of the Problem

Applying cylindrical co-ordinates [r,θ,z] with Z-axis along the axis of the circular tube, let us denote the velocity components by u,v and w along r, θ and z increasing respectively.

Let the motion is symmetrical about Z-axis. Therefore, we have $\frac{\partial}{\partial\theta} = 0$ and the nature of motion gives v=0.

The Navier-Stokes equations of motion of viscous incompressible fluid are (in absence of external forces):

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial r} + v\left[\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} - \frac{\partial^2 u}{\partial z^2}\right] \quad \text{---- (1)}$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial r} + v\left[\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} - \frac{\partial^2 w}{\partial z^2}\right] \quad \text{---- (2)}$$

The equation of continuity is

$$\frac{1}{r}\frac{\partial}{\partial r}(ur) + \frac{\partial w}{\partial z} = 0 \quad \text{---- (3)}$$

The fluid is moving along Z-axis, which shows that w is independent of z, therefore,

$$\frac{\partial w}{\partial z} = 0 \quad \text{---- (4)}$$

Using (4), the equation of continuity is:

$$\frac{1}{r}\frac{\partial}{\partial r}(ur) = 0 \ ,$$

Which gives $ur = -S$ (say), -- (5)

Where, S > 0, is the suction parameter and S < 0 is the injection parameter. Making the substitution from (5), in (1) and (2), we have:

$$\frac{1}{\rho}\frac{\partial p}{\partial r} = \frac{S^2}{r^3} \quad \text{----- (6)}$$

$$\text{And } \frac{\partial w}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + v\left[\frac{\partial^2 w}{\partial r^2} + \left(1+\frac{\delta}{v}\right)\frac{1}{r}\frac{\partial w}{\partial r}\right] \quad \text{------ (7)}$$

$$\text{Putting } \frac{\delta}{v} = 2\eta \quad \text{------ (8)}$$

$$\text{And } -\frac{1}{\rho}\frac{\partial p}{\partial r} = f(t), \quad \text{----- (9)}$$

Which is a function of alone, we get

$$\frac{1}{v}\frac{\partial w}{\partial t} = \frac{1}{v}f(t) + \left[(1+2\eta)\frac{1}{r}\frac{\partial w}{\partial r} + \frac{\partial^2 w}{\partial r^2}\right] \quad \text{------ (10)}$$

Writing $W = Fr^{-\eta}$ where W is function r and t, we get

$$\frac{\partial^2 F}{\partial r^2} + \frac{1}{r}\frac{\partial F}{\partial r} + \frac{\eta}{r^2}F = \frac{1}{v}\frac{\partial F}{\partial r} - \frac{1}{v}r^{\eta}f(t) \quad \text{----- (11)}$$

Boundary and initial conditions for the problem are:

$$w(r,t) = Fr^{-\eta} = 0 \qquad at\ r = a, t > 0$$

$$\text{And } u = -\eta_0 = \frac{S}{a} \qquad at\ r = a, t > 0 \quad \text{------- (12)}$$

$$w(r,t) = w_0(r), \quad at\ t = 0$$

$$F_0 = w_0(r)r^{\eta} \qquad at\ \ t = 0 \quad \text{----- (13)}$$

Solution of the problem:

We introduce finite Hankel transform [5] define by

$$w_H = \int_0^a rwJ_\eta(r\xi_i)dr \quad \text{----- (14)}$$

Where ξ_i is a root of the transcendental equation

$$J_\eta(a\xi_i) = 0 \quad \text{----- (15)}$$

Taking Hankel transform to equations (10) & (11), we get

$$\frac{1}{v}\frac{dF_H}{dt} + \xi_i^2 F_H = -a\xi_i F'(a)J_\eta(ax_i) + \frac{1}{v}\frac{a^{\eta+1}}{\xi_i}J_{\eta+1}(a\xi_i)f(t),$$

$$\eta > -1 \quad \text{------ (16)}$$

$$F_{H(0)}\int_0^a r^{\eta+1}w_0(r)J_\eta\,(r\xi_i)dr \quad \text{----- (17)}$$

With the help of the boundary condition (11), we get

$$\frac{dF_H}{dt} + v\xi_i^2 F_H = \frac{a^{\eta+1}}{\xi_i}vJ_{\eta+1}(a\xi_i)f(t)$$

Solution of this equation subjected to the initial condition (17) is

$$F_H = F_{H(0)}e^{-v\xi_i^2 t} + \frac{a^{\eta+1}}{\xi_i}J_{\eta+1}(a\xi_i) - \int_0^t e^{-x_i^2(t-T)}f(\tau)d\,\tau \quad \text{------- (18)}$$

Applying the inversion formula, we get

$$F(r,t) = \frac{2}{a^2}\sum_i F_{H(0)}\,e^{-v\xi_i^2 t}\,\frac{J_\eta(r\xi_i)}{[J_\eta(a\xi_i)]^2} + 2a^{\eta-1}\sum_i \frac{J_{\eta+1}(a\xi_i)}{\xi_i}\frac{J_\eta(r\xi_i)}{[J_\eta(a\xi_i)]^2}\int_0^t e^{-v\xi_i^2(t-T)}f(\tau)d\,\tau \ ,$$

$$w(r,t) = 2r^{-\eta}a^{\eta+1}\sum_i F_{H(0)}\,e^{-v\xi_i^2 t}\,\frac{J_\eta(r\xi_i)}{J_{\eta+1}^2(a\xi_i)} + 2r^{-\eta}a^{\eta-1}\sum_i \frac{J_\eta(r\xi_i)}{\xi_i J_{\eta+1}(a\xi_i)}\int_0^t e^{-v\xi_i^2(t-T)}f(\tau)d\,\tau \quad \text{----- (19)}$$

Case-I: If we take

$$w_0(r) = a^2 - r^2 \quad and \quad f(\tau) = \boldsymbol{\delta}(\tau) \ ,$$

Where $\boldsymbol{\delta}(\tau)$ is the Dirace Delta function, then

$$F(r,t) = 2a^{\eta-1}\sum_i\left[\frac{4(\eta+1)}{\xi_i^2} + 1\right]\frac{J_\eta(r\xi_i)}{J_{\eta+1}^2(a\xi_i)}\frac{e^{-v\xi_i^2 t}}{\xi_i} \ ,$$

$$w(r,t) = 2r^{-\eta}a^{\eta-1}\sum_i\left[\frac{4(\eta+1)}{\xi_i^2} + 1\right]\frac{J_\eta(r\xi_i)}{J_{\eta+1}^2(a\xi_i)}\frac{e^{-v\xi_i^2 t}}{\xi_i} \quad \text{---- (20)}$$

Taking the roots of the equation $J_\eta(a\xi_i) = 0$, $w(r,t)$ is tabulated against t.

Table-1

For different values of r (r = 0, 0.25, 0.5, 0.75) and a = 1, v = 0.1, η = 1

t	0	1	2	3	4	5	6	7	8
w(0.00,t)	1.91	0.856	0.205	0.046	0.010	0.002	0.001	0.000	0.000
w(0.25,t)	1.89	0.767	0.182	0.041	0.009	0.002	0.000	0.000	0.000
w(0.50,t)	1.61	0.533	0.124	0.028	0.006	0.001	0.000	0.000	0.000
w(0.75,t)	1.43	0.271	0.062	0.014	0.003	0.000	0.000	0.000	0.000

Table-2

For different values of r (r = 0, 0.25, 0.5, 0.75) and a = 1, v = 0.1, η = 0

t	0	1	2	3	4	5	6	7	8
w(0.00,t)	2.00	1.57	0.85	0.48	0.27	0.15	0.08	0.05	0.03
w(0.25,t)	1.94	1.33	0.74	0.42	0.23	0.13	0.07	0.04	0.02
w(0.50,t)	1.75	1.01	0.56	0.32	0.18	0.10	0.06	0.03	0.01
w(0.75,t)	1.43	0.49	0.28	0.16	0.09	0.05	0.03	0.02	0.00

Table-3
For different values of r (r = 0, 0.25, 0.5, 0.75) and a = 1, v = 0.1, η = -1/2

t	0	1	2	3	4	5	6	7	8
w(0.00,t)	2.03	1.72	1.39	1.09	0.85	0.67	0.52	0.40	0.31
w(0.25,t)	1.95	1.64	1.29	1.01	0.79	0.61	0.48	0.37	0.29
w(0.50,t)	1.76	1.30	0.99	0.77	0.60	0.47	0.37	0.28	0.22
w(0.75,t)	1.44	0.697	0.551	0.429	0.335	0.262	0.204	0.160	0.125

Case-II: If we take

$w_0(r) = a^2 - r^2 \quad and \quad f(\tau) = e^{ik\tau}$ then

$$w(r,t) = 2r^{-\eta} a^{\eta-1} \sum_i \frac{4(\eta+1)}{\xi_i^2} \frac{J_\eta(r\xi_i)}{J_{\eta+1}(a\xi_i)} - 2r^{-\eta} a^{\eta-1} \sum_i \frac{e^{-v\xi_i^2 t}}{\xi_i(ik + v\xi_i^2)} \frac{J_\eta(r\xi_i)}{J_{\eta+1}(a\xi_i)} + 2r^{-\eta} a^{\eta-1} \sum_i \frac{e^{ikt}}{\xi_i(ik + v\xi_i^2)} \frac{J_\eta(r\xi_i)}{J_{\eta+1}(a\xi_i)}$$

Using the result

$$r^{-\eta} \frac{e^{ikt}}{ik} \left[r^{-\eta} - a^\eta \frac{J_\eta\left(\sqrt{-\frac{ikr}{v}}\right)}{J_\eta\left(\sqrt{-\frac{ika}{v}}\right)} \right] = 2r^{-\eta} a^{\eta-1} \sum_i \frac{e^{ik\tau}}{\xi_i(ik + v\xi_i^2)} \frac{J_\eta(r\xi_i)}{J_{\eta+1}(a\xi_i)}$$

We get

$$w(r,t) = 2r^{-\eta} a^{\eta-1} \sum_i \frac{4(\eta+1)}{\xi_i^2} \frac{J_\eta(r\xi_i)}{J_{\eta+1}(a\xi_i)} - 2r^{-\eta} a^{\eta-1} \sum_i \frac{e^{-v\xi_i^2 t}}{\xi_i(ik + v\xi_i^2)} \frac{J_\eta(r\xi_i)}{J_{\eta+1}(a\xi_i)} + r^{-\eta} \frac{e^{ikt}}{ik} \left[r^\eta - a^\eta \frac{J_\eta\left(\sqrt{-\frac{ikr}{v}}\right)}{J_\eta\left(\sqrt{-\frac{ika}{v}}\right)} \right]$$

If there is no arbitrary initial velocity distribution, then the real part of the solution:

$$R_e(w) = R_e \left[r^{-\eta} \frac{e^{ikt}}{ik} \left\{ r^\eta - a^\eta \frac{J_\eta\left(\sqrt{-\frac{ikr}{v}}\right)}{J_\eta\left(\sqrt{-\frac{ika}{v}}\right)} \right\} \right]$$

Is a particular solution of the problem of viscous incompressible fluid through a straight porous circular tube due to the influence of pressure gradient which is the same as obtained by [6]. When η = 0

If we consider the small oscillation of highly viscous fluid, the quantity $\sqrt{-\frac{ik}{v}}$ becomes very small.

Expanding the Bessel's function in power series and neglecting the terms $O(v^{-2})$, we get:

$$w(r,t) = \frac{e^{ikt}}{ik} \left[1 - \left\{ 1 + \frac{ikr^2}{4v(\eta+1)} \right\} \left\{ 1 - \frac{ika^2}{4v(\eta+1)} \right\} \right] = \frac{e^{ikt}}{4v(\eta+1)} (a^2 - r^2)$$

This corresponds to the steady flow of viscous liquid through a straight porous channel due to a periodic pressure gradient.

From the equation (20) it follows that due to the application of impulsive pressure gradient the velocity profile dies out exponentially as t increases and ultimately tends to zero when $t \to \infty$.

Conclusion

Table (1) shows the distribution of velocity when suction parameters is equal to 2v and Table (2) shows the same when the suction parameter is absent. A comparative study of the two tables reveals the fact that suction plays a role for destroying the motion while Table (2) and (3) show the injection accelerates the motion, but the effect of suction is more prominent than that of injection at the beginning of the motion.

Acknowledgement

Author is thankful to Prof.(Dr). D.S.Negi, Dept. of Mathematics, HNB Garhwal University, Srinagar, Uttarakhand for giving permission for publishing this manuscript during the Ph.D work.

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